IZS G. CAPORALE

OIE

Collaborating Centre
for Veterinary Training,
Epidemiology, Food Safety
and Animal Welfare

Assessing Convergence of MCMC algorithms

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To implement a MCMC method

Important issues are:

- the choice of sampler
  - Gibbs sampler or
  - Metropolis Hastings Algorithms

  are the most famous
- the number of independent replications to be run

To implement a MCMC method

Important issues are:

- the choice of starting values
  - Usually iterates within an initial transient phase or *burn in* period are discarded
  - But how long should the *burn in* period be?
  - Rates of convergence of algorithms on different targeted distributions vary considerably
- estimation and efficiency problems
  - Computational expense
  - Easily to implement
  - Easily to interpreter.

To implement a MCMC method

Ideally we would like to analytically compute or estimate the Markov chain convergence rate and then take sufficient iterations to satisfy any prescribed accuracy criteria.

However this is not possible in general.

To implement a MCMC method

- But we can use specialized techniques to *bound* convergence rates for MCMC.
- Or we can carry out some form of statistical analysis in order to assess convergence.
- There are also a number of methods which attempt to measure the performance of any particular sampler.

Convergence Diagnostics

- It is a method for assessing how long to run a Markov chain in order to obtain observations from (or approximately from) the stationary distribution.

The Yu and Mykland's CUSUM method

- The CUSUM method is a graphical method which monitors convergence via a CUSUM plot based upon the sampler output.
- It was proposed by Yu and Mykland (1997).
- It can be applied to any sampler,
- and can be implemented by generic problem-independent code
- It has been integrated with an objective measure of convergence.

The Yu and Mykland's CUSUM method

We will repeat all the steps for each variable Y at a time (prevalence 1, 2, Sensitivity 1, 2 and Specificity 1,2)

- Given the output $\{Y_1, \ldots, Y_n\}$
  - we begin by discarding the initial n_0 iterations which we believe to correspond to the *burn in* period
  - We then construct CUSUM path for a scalar summary statistic,
  - as follows:

The Yu and Mykland's CUSUM method

- Step 1

Calculate the mean value of all calculated values from n_0 to n for variable Y

$$\hat{\mu} = \frac{1}{n - n_0} \sum_{t=n_0+1}^{n} Y_t$$

The Yu and Mykland's CUSUM method

- Step 2

Calculate the CUSUM or partial sum

$$S_T = \sum_{t=n_0+1}^{T} \left(Y_t - \hat{\mu} \right) \qquad for\ T = n_0 + 1, \ldots, n$$

The Yu and Mykland's CUSUM method

- Step 3

Plot $\{\hat{S}_T\}$ against T for $T=n_0+1, \ldots, n$ connecting successive points by line segments.

The Yu and Mykland's CUSUM method

- Step 4

Define the "hairiness" of the plot, as:

$$d_T = \begin{cases} 1 \quad if\ S_{T-1} > S_T\ and\ S_T < S_{T+1} \\ or\ S_{T-1} < S_T\ and\ S_T > S_{T+1} \\ 0 \quad else \end{cases}$$

for $T=n_0+1, \ldots, n-1$

d_T takes values between 0 and 1, where 0 indicates a smooth plot and a value of 1, indicates maximum "hairiness".

The Yu and Mykland's CUSUM method

- Step 5

The index:

$$D_{n_0,n} = \frac{1}{n-n_0} \sum_{T=n_0+1}^{n-1} d_T$$

is a binomial outcome with mean ½ and variance 1/ 4(n-n_0)

The Yu and Mykland's CUSUM method

- Step 6

Calculate the confidence limits

$$\tfrac{1}{2} \pm 1.96\sqrt{\frac{1}{4(n-n_0)}}$$

The Yu and Mykland's CUSUM method - Example

Initial data

	A	B	C	D	E	F	G	H	I	J
1										
2										
3	**population 1**	T2+	T2-			**population 2**	T2+	T2-		
4	T1+	15	3	**18**		T1+	10	2	**12**	
5	T1-	1	129	**130**		T1-	0	150	**150**	
6		**16**	**132**	**148**			**10**	**152**	**162**	
7										
8										